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INFLUENCE OF GRAIN SIZE ON FATIGUE NOTCH-SENSITIVITY*

BY R. W. KARRY[1] AND T. J. DOLAN[2]

SYNOPSIS

A study was made of the reduction in flexural fatigue strength caused by a circumferential notch in specimens of a single-phase metal. Groups of specimens were prepared from 70-30 brass having the same chemical analysis but processed to obtain three different degrees of cold reduction and three different annealed grain sizes. Two notch shapes having the same depth but different root radii were investigated. It was found that the fatigue strength reduction factor and notch-sensitivity were not unique material constants; these were dependent upon both the grain size of the metal and the notch root radius. An increase in grain size produced a significant lowering of the fatigue strength, but also resulted in a decrease of notch-sensitivity and fatigue strength reduction factor. Experimentally determined fatigue strength reduction factors were somewhat smaller than those computed from H. Neuber's theory of the effect of an elementary particle on notched material behavior. On comparing fatigue strengths developed, a correlation was found between the calculated average stress on a grain located in the critically stressed surface of a notched specimen and that in an unnotched specimen having the same grain size.

One of the most important problems encountered in the design of machine parts is the prediction of the strength of a member when subjected to the stress concentrating effects developed by a geometrical notch. Several experimental investigations (**4, 5, 12**)[3] have disclosed that the fatigue limit or fatigue strength of notched fatigue specimens is usually greater than that predicted by use of theoretical stress concentration factors based on assumptions of elasticity. Several expressions have been developed in attempts to explain this discrepancy between classical theory and experimental results. Among these expressions, the two which have come to be commonly accepted[4] are the "fatigue strength reduction factor," K_f, and the "notch-sensitivity," q. Notch-sensitivity supposedly varies between zero, when $K_f = 1$, and unity, when $K_f = K_t$; however, values of q less than zero (negative) and greater than one have been observed in tests of several different materials. Neither K_f nor q have any significance unless the stress range, geometry, size, and material of the specimen are stated.

For conditions of *static* loading, Neuber (**3**) has discussed the failure of the classical theory of elasticity to predict adequately the behavior of metals with

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[3] The boldface numbers in parentheses refer to the list of references appended to this paper, see p. 13.

[4] K_f is defined as the ratio of the fatigue strength of specimens with no stress concentration to the fatigue strength of members with a stress raiser. Notch sensitivity, q, is a measure of the degree of agreement between K_f and K_t, where K_t is the theoretical stress concentration factor. The value of q is defined (**10**) as follows:

$$q = \frac{K_f - 1}{K_t - 1}$$

notches of very small radii. The theory of elasticity assumes the material to be perfectly homogeneous and made up of infinitesimally small particles which is *not* the case for engineering materials. If the radius of curvature of the surface is everywhere large enough, compared with the size of the crystalline structure of the material, no pronounced error in the classical theory becomes evident. To take into consideration, mathematically, the crystalline structure of metals, Neuber assumed the material to be composed of numerous small but finite structural units which he named the "elementary particle." The values of theoretical stresses in the region of peak stress were averaged over the surface of the elementary particle which reduced the value of the effective maximum elastic stress because of the stress gradient existing over the particle. Consequently, Neuber found that the stress concentration factor also depends upon the size of the elementary particle when the surface curvature is relatively sharp. It appears possible that this more realistic approach to the stress concentrating effect of a sharp notch in a polycrystalline metal may also be applicable to the case of repeated loading.

The work of Peterson (1, 2) lends some support to this suggestion, at least in a qualitative manner. Briefly, Peterson found that fine-grained, heat-treated steels were more notch sensitive in fatigue than coarse-grained, annealed steels when tested under similar conditions. Results obtained by other investigators (8, 11) also seem to indicate a possible relationship between metallurgical structure and fatigue notch sensitivity. One might reason that a stress raiser introduces a steep stress gradient at the specimen surface, resulting in an appreciable stress drop across any one grain assumed to be located at the root of the notch. For metal having large grains, there would be a relatively large stress drop and, hence, a lower average stress acting on the grain. Thus for the same *peak* stress calculated at the surface, the large grains are subjected to a less severe loading condition. Therefore the fatigue behavior of a notched specimen could not be expected to be the same as that of an unnotched specimen where the drop in stress across one grain is relatively small.

Although the contributions discussed above are of considerable importance, some confusion exists concerning what measure of grain size was utilized (1, 2) or if the elementary particle (3) is related to the size of a metal space lattice or to some kind of crystal grain.

A survey of the literature on fatigue notch sensitivity did not disclose any instances in which a systematic investigation of the influence of grain size was made. In view of the lack of available data on this subject, it seemed desirable that a laboratory study be made to furnish the needed information.

PURPOSE AND SCOPE

The purpose of this investigation was to study the influence of measured grain sizes on the fatigue notch sensitivity of a metal. In order to keep the number of unknown variables to a minimum, it was thought advisable to select for test a simple single-phase metal in which the grain size could readily be controlled; for this reason, a 70-30 brass was used in the experimental program. The effects of three different degrees of cold work and three different annealed grain sizes on the notch sensitivity of the 70-30 brass were determined experimentally. Notched fatigue specimens were tested at approximately 3500 rpm in rotating cantilever-beam machines under constant load at zero mean stress. Two different notch shapes were investigated to reveal the effect of notch root radius on the notch-sensitivity of the brass.

In addition, Neuber's theory of the effect of an elementary particle on notched material behavior was investigated for its applicability to the case of repeated loading.

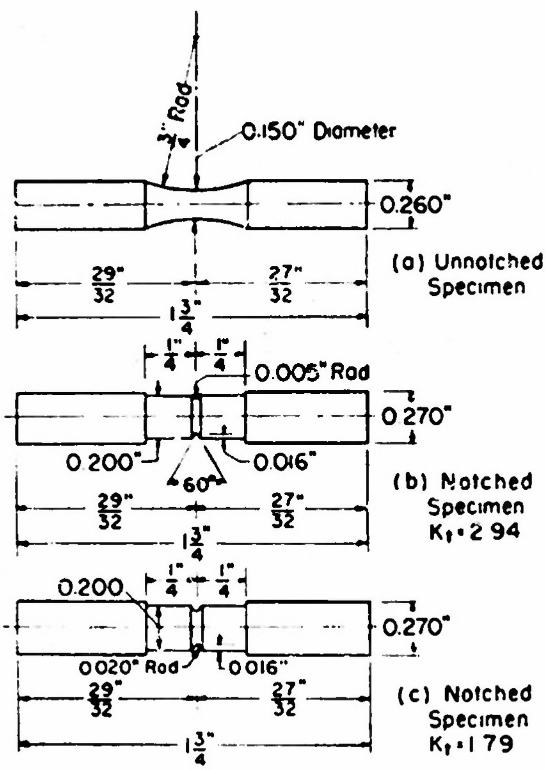


FIG. 1.—Details of Fatigue Test Specimens.

MATERIAL AND PREPARATION OF SPECIMENS

The brass was received in the form of cold-drawn rod and was from the same heat and processing as that used in a previous investigation (13). It was composed of 69.5 per cent copper, less than 0.05 per cent lead, 0.01 per cent iron, and the balance zinc. The various specimen groups were subjected to different mechanical treatments by the manufacturer as listed in Table I.

Notched fatigue specimens, Figs. 1(*b*) and (*c*), were first machined from groups of the "as-received" material and some of these groups were later given specific annealing treatments to alter the grain size. The theoretical stress concentration factors, K_t, were determined by means of Neuber's nomograph (3). The unnotched fatigue specimen, Fig. 1(*a*) was used by Sinclair and Craig (13); it is included in this paper since the unnotched fatigue strengths previously determined by Sinclair and Craig for specimens from the same groups of bar stock were utilized in analyzing the results.

TABLE I.—"EQUIVALENT" GRAIN SIZES OF COLD-DRAWN BRASS.

Group Designation	Ready-to-Finish Grain Size, mm	Cold Reduction in Area, per cent	Final Diameter of Rod, in.	Equivalent Grain Size, mm
A	0.015	20	0.324	0.00295
D	0.015	40	0.281	0.00205
G	0.015	60	0.229	0.00185

In the case of cold-worked metals, it is practically impossible to determine the grain size of the material by using optical methods. However by introducing a new concept, Sinclair and Craig (13) determined an "equivalent" grain size for cold-worked brass. They determined the mean grain diameters, by Jeffries' planimetric method,[5] and Rockwell hardness numbers of specimens subjected to various annealing treatments. These values are plotted as solid black dots in Fig. 2. It had previously been shown (7) that a straight-line relation appears to exist between the log of the mean grain diameter and the Rockwell hardness of annealed brass. Sinclair and Craig reasoned that if the Rockwell hardness number of a cold-worked brass is known, an "equivalent" grain diameter

[5] Tentative Methods for Estimating the Average Grain Size of Wrought Copper and Copper-Base Alloys (E 79 - 49 T), 1952 Book of ASTM Standards, Part 2, p. 1146.

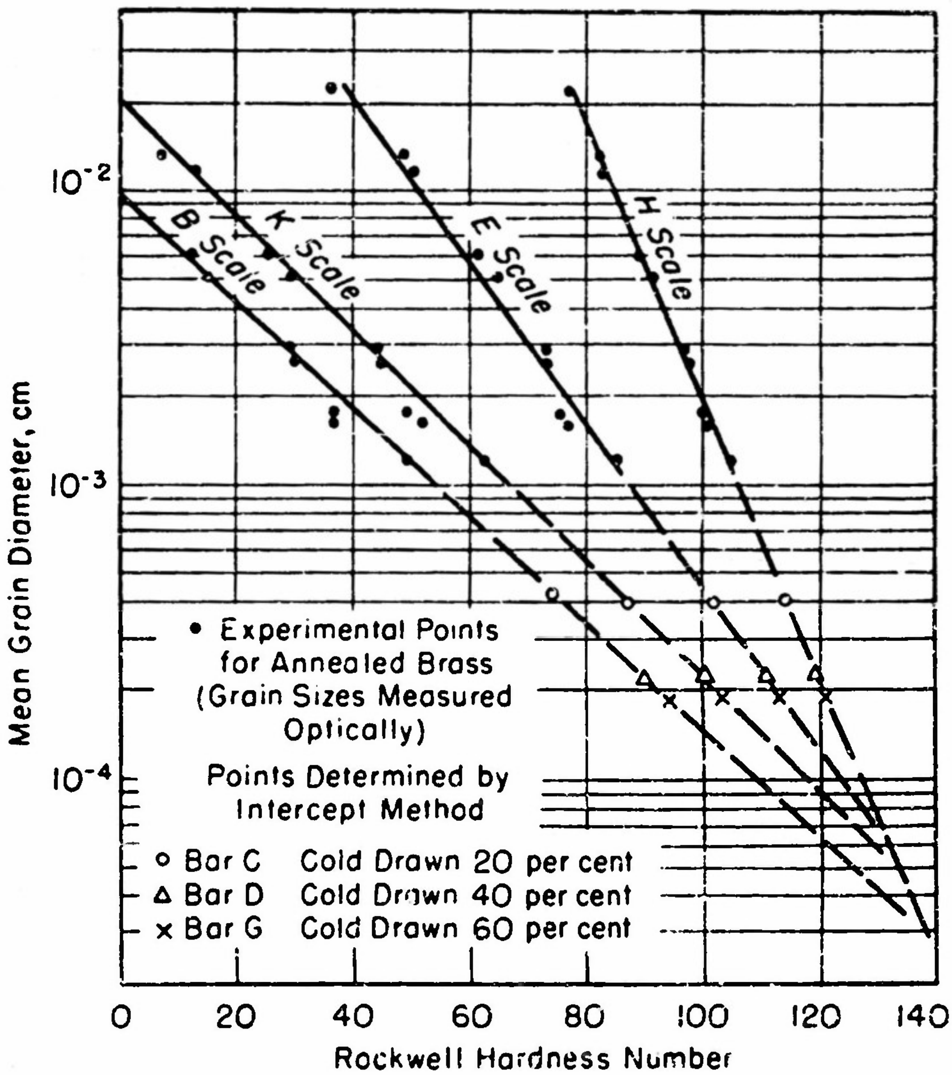


FIG. 2.—Effect of Mean Grain Diameter on Rockwell Hardness Number.
Each point is an average value for at least ten hardness readings. From Sinclair and Craig (13).

TABLE II.—ANNEALING TREATMENTS AND GRAIN SIZES OF SPECIMENS.

Stress Concentration Factor, K_t	Group Designation	Anneal Temperature, deg Fahr	Time at Temperature, hr	Approximate Annealed Grain Size, mm
2.94	G	750	4	0.0125
	A	960	4	0.033
	A	1200	4	0.125
1.79	A	960	4	0.023
	A	1200	4	0.107
	A	1400	2	0.28

TABLE III.—UNNOTCHED FATIGUE STRENGTHS AT 100 MILLION CYCLES.[a]

Brass Group	"Equivalent" or Mean Grain Size, mm	Fatigue Strength at 10^8 Cycles, psi
G	0.00185	40 000
D	0.00205	30 000
A	0.00295	24 000
Anneal G	0.012	22 000
Anneal A	0.026	17 000
Anneal A	0.131	12 000
Anneal A	0.305	9 000

[a] From Sinclair and Craig (18).

may be established by the intercept method on an extrapolation of the curves in Fig. 2. These estimated grain sizes were observed to be "equivalent" as far as their general effects on the mechanical properties of the brass were concerned. The term grain size as used throughout the remainder of this paper will refer to the "equivalent" grain sizes or cold-worked brass and the mean grain sizes (Jeffries' method) for annealed brass. The grain sizes of the as-received groups studied are given in Table I.

TABLE IV.—NOTCHED FATIGUE STRENGTHS AT 100 MILLION CYCLES.

Stress Concentration Factor, K_t	"Equivalent" or Mean Grain Size, mm	Fatigue Strength at 10^8 Cycles, psi
2.94	0.0018	13 000
	0.0020	12 000
	0.0029	12 000
	0.012	12 000
	0.033	10 000
	0.125	9 000
1.79	0.0018	23 000
	0.0020	21 000
	0.0029	16 000
	0.023	12 000
	0.107	10 000
	0.28	9 000

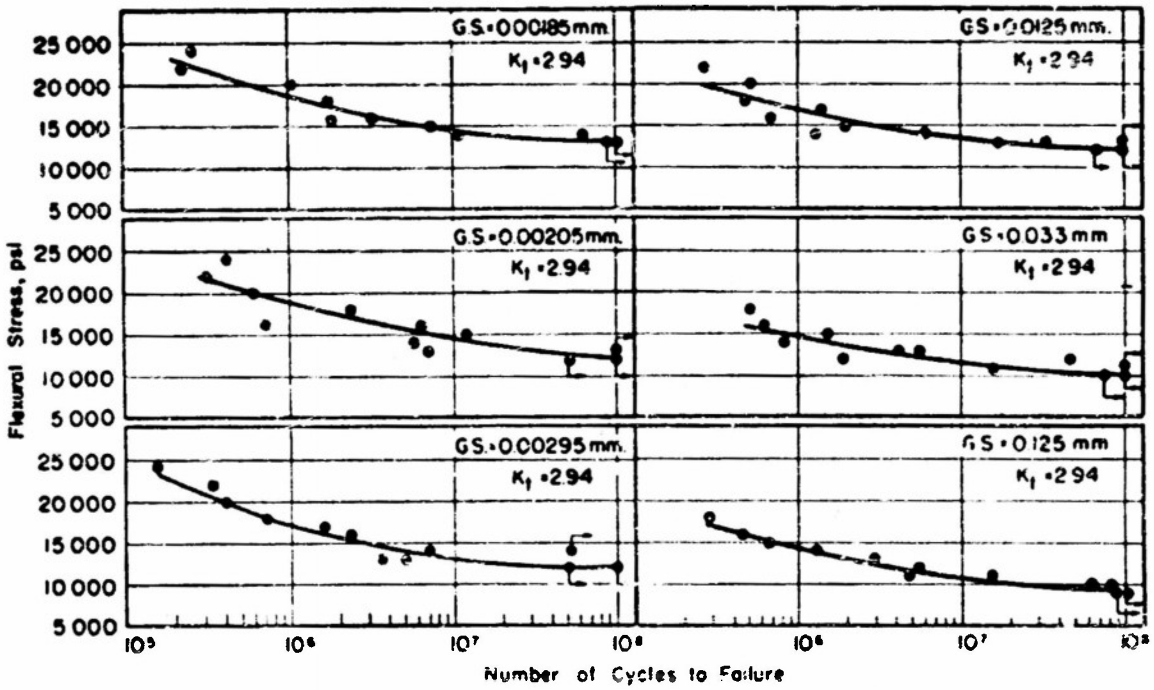
After machining, a number of notched specimens were given various annealing treatments as listed in Table II. The annealed grain sizes resulting from these treatments were also determined from the hardness-grain size relation of Fig. 2. The annealed grain sizes for specimens of each notch are given in Tables II to IV. The differences in grain size for the same annealing treatment resulted from heating the batches of specimens of each notch separately. The Rockwell hardness number of each group given an annealing treatment was the average value of at least 20 hardness readings. These hardness readings were obtained on specimens which were approximately 1¾ in. in length and which had flats milled on two sides of the rod. The hardness readings for the various groups agreed closely with the data plotted in Fig. 2 for the different grain sizes investigated.

The test section of each specimen was polished electrolytically, using a solution of 530 g per l of orthophosphoric acid in water, at a temperature of 100 ± 1 F for a period of 10 min as the final step preparatory to testing. The electropolishing removed approximately 0.001 in. of metal from the diameter, uniformly along the length of the test section.

TABLE V.—INFLUENCE OF "EQUIVALENT" OR MEAN GRAIN SIZE ON THE FATIGUE STRENGTH REDUCTION FACTOR AT VARIOUS NUMBERS OF CYCLES.

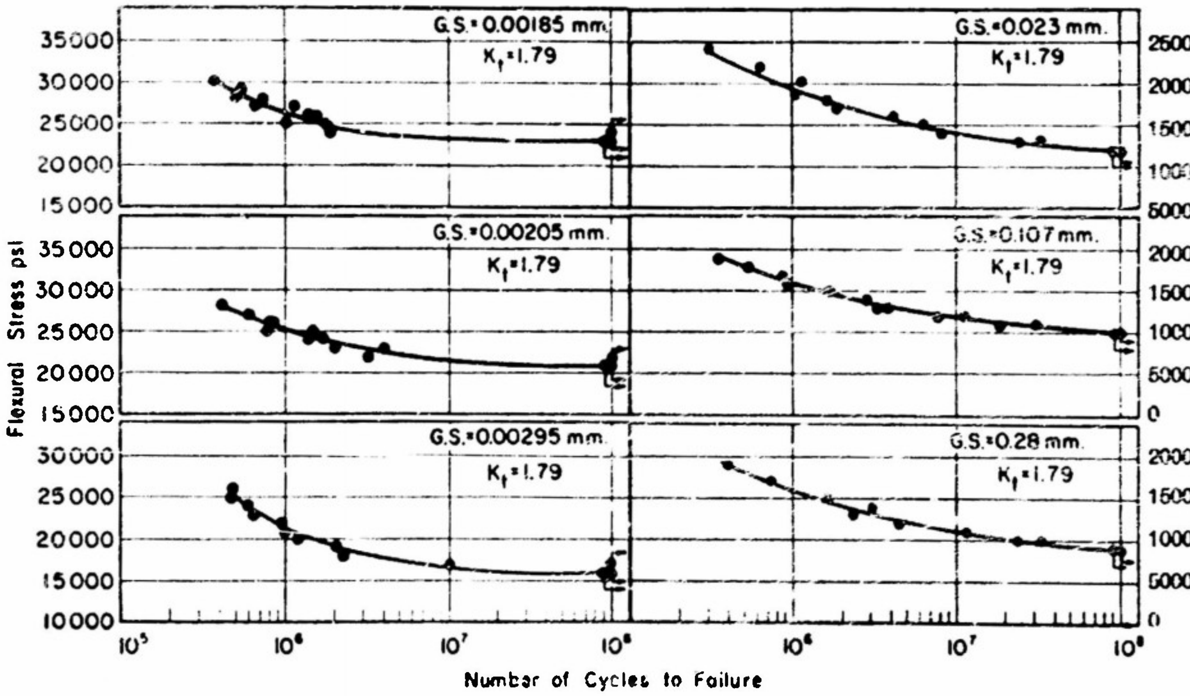
Stress Concentration Factor, K_t	"Equivalent" or Mean Grain Size, mm	Fatigue Strength Reduction Factor, K_f — Number of Cycles: 5×10^5	10^6	5×10	10^7	5×10^7	10^8
2.94	0.00185		2.86	2.76	2.84	3.01	3.10
	0.00205	2.54	2.46	2.19	2.19	2.29	2.46
	0.00295	2.01	1.97	1.85	1.92	2.06	2.12
	0.0125	1.85	1.83	1.72	1.73	1.89	1.82
	0.033	1.75	.72	1.63	1.59	1.66	1.60
	0.125	1.61	1.54	1.46	1.37	1.27	1.33
1.79	0.00185		2.12	1.84	1.79	1.74	1.70
	0.00205	1.88	1.84	1.55	1.51	1.47	1.56
	0.00295	1.55	1.58	1.48	1.53	1.52	1.57
	0.023	1.42	1.44	1.42	1.42	1.44	1.46
	0.107	1.34	1.34	1.31	1.28	1.20	1.28
	0.28						1.04

After polishing, all specimens were measured on three different diameters, and the notch contour checked at 50× on an optical microprojector. For any one specimen the maximum difference in diameters was observed to be less than 0.001 in. For a majority of the specimens this difference was observed to be less than 0.0005 in. The average value of the specimen diameters at the root of the notch was used to calculate the nominal stress in the specimen by the elementary flexure formula, $S = \frac{Mc}{I}$.



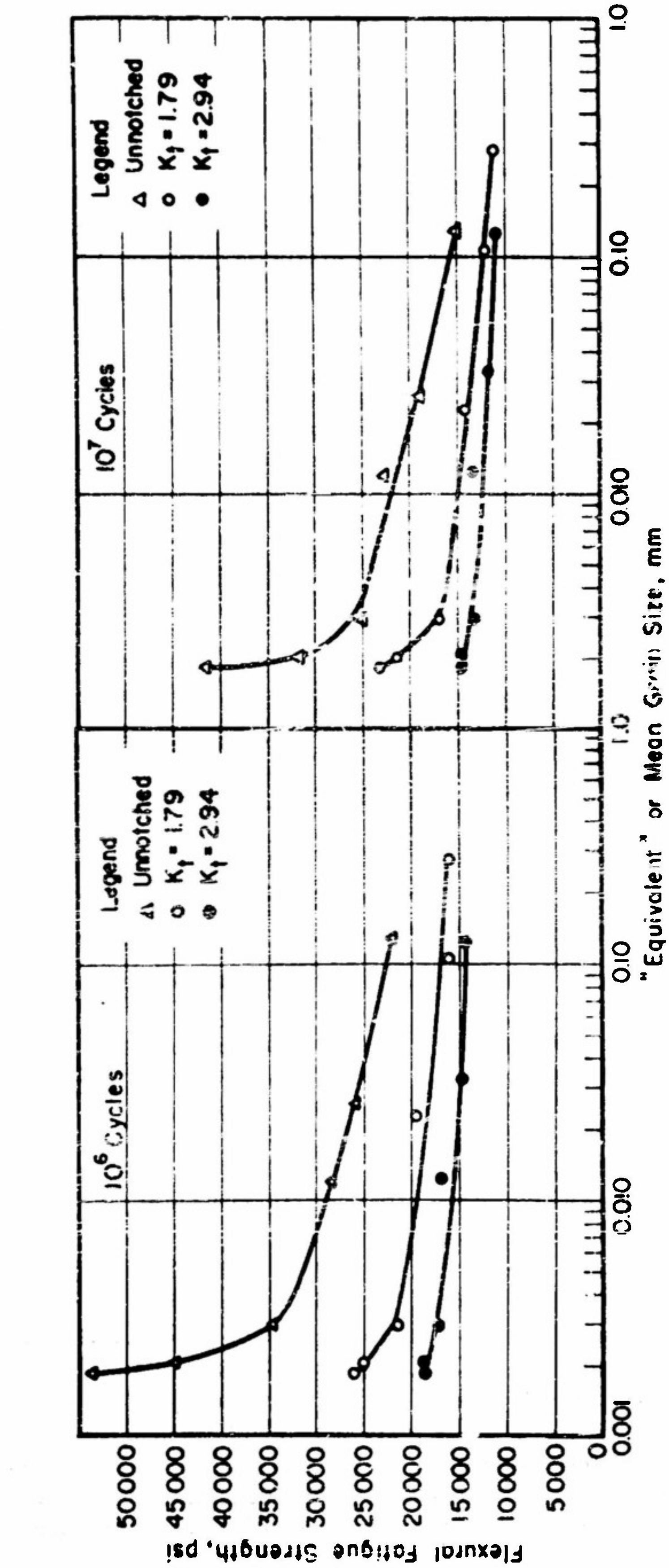
(*a*) "Equivalent" grain size. (*b*) Annealed grain size.

FIG. 3.—Influence of "Equivalent" and Annealed Grain Size on Fatigue Strength of 70-30 Brass.



(*a*) "Equivalent" grain size. (*b*) Annealed grain size.

FIG. 4.—Influence of "Equivalent" and Annealed Grain Size on Fatigue Strength of 70-30 Brass.



(a) At 10^6 cycles.

(b) At 10^7 cycles.

FIG. 5.—Influence of "Equivalent" or Mean Grain Size on the Fatigue Strength.

Results of Tests and Discussion

The fatigue strengths at 100 million cycles for the unnotched brass having various grain sizes are given in Table III.

Specimens with either type of notch (see Figs. 1(*b*) and (*c*)) were loaded as rotating cantilever beams. The data obtained are plotted in the typical S-log N diagrams shown in Figs. 3 and 4. The fatigue strengths at 100 million cycles for specimens with each notch and for the various grain sizes investigated are listed in Table IV.

For purposes of quantitative comparison, the fatigue strength for any given number of stress cycles can be obtained from the curves of Figs. 3 and 4 for each grain size investigated. These values have been replotted as a function of material grain size in Figs. 5 and 6 for various numbers of stress cycles. From these curves it can be seen that the fatigue strength of the material at each selected life increased as the grain diameter was reduced.

By scaling data from the smooth curves of Figs. 5 and 6, the fatigue strength reduction factors, K_f, were computed for each grain size at each selected life, and the values listed in Table V. From a comparison of these data, it can be seen that for a given life, K_f increased as the grain size decreased. It is also important to note that for any one grain size, K_f in general was *slightly larger* when high loads were employed to produce failure at lower numbers of cycles. This is contrary to the popular belief that higher average stresses cause a localized redistribution of stress that may lower the strength reduction factor.

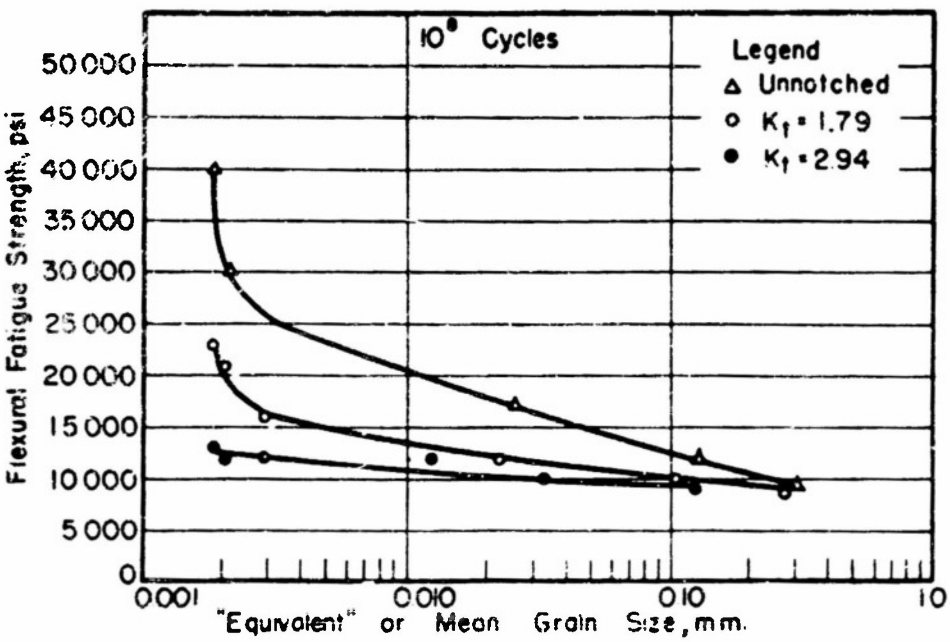


FIG. 6.—Influence of "Equivalent" or Mean Grain Size on the Fatigue Strength at 10^8 Cycles

The fatigue strength reduction factors, K_f (as computed from the curves shown in Fig. 6) and the fatigue notch-sensitivities, q, are plotted *versus* the grain size of the brass in Fig. 7. It can be seen from these illustrations that K_f and q are not unique material constants but are dependent upon both the grain size of the metal and the notch root radius. In addition, it can be observed that as the grain diameter is reduced, K_f and

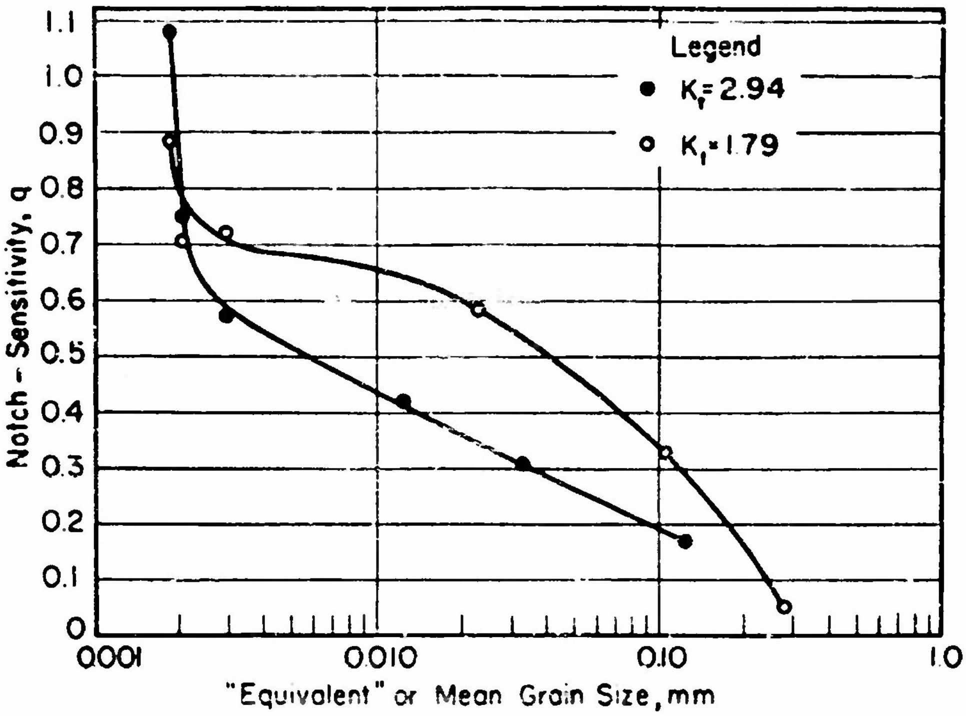


FIG. 7.—Influence of "Equivalent" or Mean Grain Size on Notch Sensitivity at 10^8 Cycles.

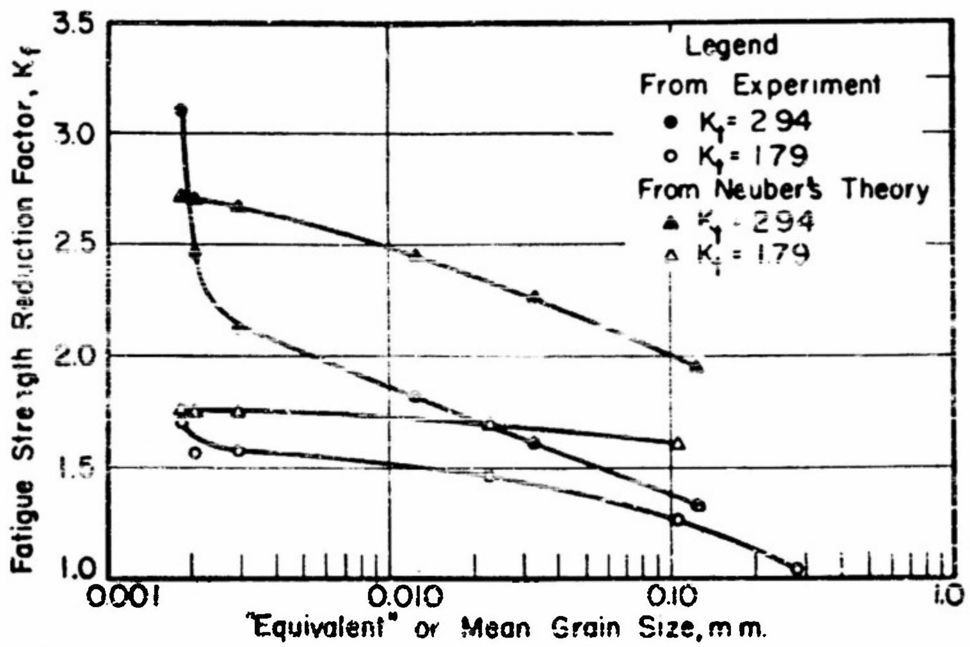


FIG. 8.—Fatigue Strength Reduction Factors at 10^8 Cycles Compared with Neuber's Theory of Notched Material Behavior.

q for both notches more nearly approach the values predicted by the theoretical stress concentration factors, K_t. This behavior is in agreement with that observed by Peterson (1, 2). The stress gradient set up in the specimen by the notch, results in an appreciable stress drop across any grain located at the root of the notch. There would be a greater drop in stress across a large grain (and hence the average stress would be smaller) than for a small grain size. For the effective stress concentration factor, according to Neuber, is given by the following expression:

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{\frac{\rho'}{r}}} \quad \text{.......(1)}$$

where:

K_f and K_t = previously defined,
r = notch radius, and
ρ' = half the width of the elementary particle.

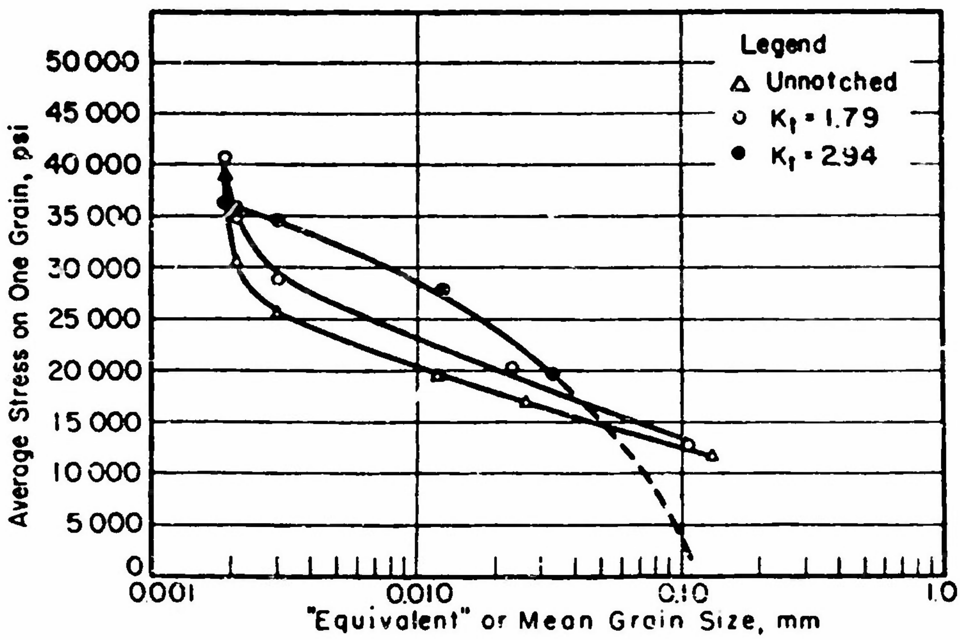


FIG. 9.—Average Stress on One Grain Located in the Critically Stressed Surface of a Notched or Unnotched Specimen (Based on 10^8 Cycles of Stress).

the same maximum (or peak) stress, a fine grain located in the critical zone is therefore subjected to a more severe condition of stressing.

It would be of considerable aid in formulating a design procedure if a rational basis for predicting K_f and q was available. The results of Neuber's (3) analysis of the effect of an elementary particle on notched material behavior, for the condition of static loading, were investigated for their applicability to this problem. When the surface curvature of a member is relatively sharp, Neuber also found that for relatively sharper notches, the notch flank angle, ω, had to be considered. The effect of the notch flank angle was found to be satisfied by the following expression:

$$K_f = 1 + \frac{K_t - 1}{1 + \frac{\pi}{\pi - \omega}\sqrt{\frac{\rho'}{r}}} \quad \text{.....(2)}$$

The liberty of using fatigue symbols in Neuber's expressions has been taken since his static theory was applied to the case of repeated loading.

If the grain diameter of the brass is assumed to be the same as Neuber's concept of an elementary particle, Eqs 1 and 2 yield the curves shown in Fig. 8 for 100 million cycles of stress. The values of K_f for both notches as calculated from Fig. 6 have also been included for comparison. One-half the grain diameter was assumed to be equal to ρ', which is half the width of Neuber's elementary particle. Equation 2 was used for the notch having a root radius of 0.005 in. and a flank angle of 60 deg, whereas Eq 1 was applied for the notch having a root radius of 0.020 in. From the curves in Fig. 8 it can be seen that Neuber's theory predicts fatigue strength reduction factors which are considerably higher than the experimental values for all but the smallest grain sizes considered. Further, it can be seen that there is a greater difference between the theoretical and experimental values for the notch having a root radius of 0.005 in. than the results for the notch with a root radius of 0.020 in. It must be remembered, however, that Neuber considered the material to be subjected to static loads.

Average Stress on Critically Stressed Grain

It was thought that some correlation might exist between the average stress on a grain located in the surface of an unnotched specimen and that acting on a grain located at the root of the notch of a notched specimen. The stress gradient in the unnotched specimen was determined from the elementary flexure formula. The stress gradients set up by the two notches were calculated from an expression approximated by Seely and Smith (14) from Neuber's work. This expression is as follows:

$$\frac{S}{K_t S_n} = \frac{2.5}{\rho} + \frac{1}{c} \qquad (3)$$

where:

S = maximum stress gradient, psi per in.,

$K_t S_n$ = theoretical maximum stress, where S_n is the nominal stress as determined by elementary flexure formula, psi,

ρ = notch root radius, in., and

c = radius of section at root of notch, in.

This expression yields the maximum stress gradient at the root of the notch of a shallow-grooved bar under bending. The criterion for a groove being shallow is that the depth of the groove be less than one-half the radius of the net section at the notch.

The average stress on a grain located at the root of either notch was computed in the following manner. The maximum stress on the grain was assumed to be equal to the product of the nominal fatigue strength (for that particular grain size) and the theoretical stress concentration factor, K_t. The maximum stress gradient in the specimen was computed for the given notch (by Eq 3) and the decrease in stress across the grain was approximated by multiplying the grain diameter by the stress gradient. The minimum stress on the grain could thus be approximated by taking the difference between the maximum stress and this decrease in stress; the approximate average stress was then computed by taking one-half the sum of the maximum and minimum stresses on the grain. In each case the fatigue strength for a particular grain size was obtained from the smooth curves of Fig. 6.

The average stress on a grain located at the root of the notch is compared in Fig. 9 with the average stress on a grain in the surface of an unnotched specimen for each grain size investigated. It can be seen in Fig. 9 that the average stresses on grains in the unnotched specimens and in the specimens having a theoretical concentration factor K_t = 1.79 are in rather close agreement for all grain sizes considered. However, in the case of the notch having a root radius of 0.005 in. (K_t = 2.94), considerable deviation is observed for grain sizes greater than about 0.03 mm. This small root radius sets up an extremely steep stress gradient over a very shallow depth below the root of the notch, and the slope to the stress distribution curve changes markedly across the specimen. Thus for the larger grain sizes, the arbitrary assumption of a constant stress gradient across one grain is greatly in error. For example, with the largest grain diameter, 0.125 mm, the decrease in stress across the grain (as predicted by assuming a constant stress gradient) is so large that a large compressive stress would be necessary at the bottom of the grain. Hence, for this sharp notch, the stress gradient as calculated from Eq 3, in the manner described above, introduces considerable error in calculating the values for the average stresses on the larger grain sizes.

CONCLUSIONS

By means of flexural fatigue tests the notch sensitivity of a typical single-phase metal was investigated for a wide range of grain sizes and for specimens with two different stress raisers. As a result of this study, the following conclusions seem justified:

1. A *decrease* in the grain size (or of "equivalent grain diameter" of cold-rolled brass) resulted in the following observed change in properties:

(*a*) The fatigue strength of both notched and unnotched specimens was increased.

(*b*) For notched specimens the fatigue strength reduction factor, K_f, and the notch-sensitivity, q, were both increased.

(*c*) For the finest grain sizes studied the values of K_f and q tended to approach the values predicted by the theoretical stress concentration factors, K_t.

2. In general, for any given grain size, the fatigue strength reduction factor and the notch-sensitivity were slightly *larger* for the higher stress levels that produced failure at low values of fatigue life.

3. The fatigue strength reduction factor and notch sensitivity are *not* unique material constants; their values depend upon both the notch root radius and the grain size of the metal.

4. A fair correlation was shown to exist between the calculated *average* stress on the most highly stressed grain in a notched specimen with that calculated for an unnotched specimen of the same grain size.

Acknowledgment:

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